20th IMEKO TC10 Conference
"Measurement for Diagnostics, Optimization and Control to Support Competitiveness and Innovation"
Lisbon, Portugal, September 28–29, 2026

LPV Feedforward Iterative Tuning for a Cam-Driven Precision Motion Stage

Li Li[1], Song Shuo[1], Dong Yue[1], Li Liyi[2]

[1] *Center of Ultra-precision Optoelectronic Instrument Engineering, Harbin Institute of Technology, Harbin, China, hitlili@hit.edu.cn, 23b901017@stu.hit.edu.cn, ydonghit@hit.edu.cn*
[2] *Department of Electrical Engineering, Harbin Institute of Technology, Harbin, China, liliyi@hit.edu.cn*

***Abstract* – This paper proposes a data-based Linear Parameter-Varying (LPV) feedforward iterative tuning method for a cam-driven precision motion stage with position-dependent nonlinear gain. To overcome the limitations of standard Linear Time-Invariant (LTI) control method, the proposed feedforward controller is parameterized using position-velocity coupled polynomial basis functions. Embedding high-order position variables into the velocity channel creates spatially varying basis functions that inherently capture the position-dependent LPV dynamics. The effectiveness of the proposed method is validated via experimental results.**

Keywords – Precision motion stage, LPV, Feedforward iterative tuning, Motion control.

I. INTRODUCTION

Cam-driven micro-positioning stages are widely used in precision manufacturing applications, such as semiconductor lithography, microscopy, and optical inspection, due to their compact structure, high stiffness, and excellent force density [1]. By converting continuous rotary motion into precise linear displacement, eccentric cam mechanisms provide an effective solution for high-load precision actuation. However, the input is applied in the rotational domain while the output is measured in the translational domain, resulting in a non-collocated input–output relationship and a nonlinear mapping between actuation and measurement.

From a control perspective, the resulting nonlinear mapping leads to a position-dependent gain, implying that the control system can no longer be characterized as Linear Time-Invariant (LTI), but rather exhibits Linear Parameter-Varying (LPV) characteristic [2,3]. Thus, LTI control methods struggle to maintain consistent stability margins and tracking performance throughout the entire stroke, leading to significant dynamic tracking errors [4].

To achieve higher tracking accuracy, feedforward control methods are widely adopted. In standard LTI systems, the feedforward controller is typically designed as constant gains proportional to the reference velocity and acceleration [5]. However, applying such a LTI feedforward strategy directly to LPV systems is fundamentally inadequate, because the required amount of compensation varies with the operating point. Furthermore, although an accurate analytical model of the cam mechanism could in principle eliminate this nonlinearity, such an analytical model is often not fully applicable due to various installation deviations [6].

In practice, to avoid using accurate model information, data-driven methods are widely used [7-10]. While iterative learning control effectively suppresses repetitive disturbances, it lacks generalization to non-repetitive reference trajectories [8]. To address this limitation, feedforward iterative tuning can be used to optimize the parameters of a predefined feedforward controller [9,10].

In this paper, a data-based LPV feedforward iterative tuning method is proposed for a cam-driven precision motion stage. The feedforward controller is parameterized using position–velocity coupled polynomial basis functions, enabling effective compensation of the position-dependent transmission gain without requiring an accurate analytical model. In contrast to conventional velocity feedforward which uses constant gains, the proposed method adapts the feedforward signal according to the motion position.

The remainder of this paper is organized as follows: Section II derives the kinematic model and illustrates the LPV characteristic of the stage. Section III details the LPV feedforward iterative tuning method. Section IV presents the experimental setup and comparative results. Finally, Section V concludes the paper.

II. PROBLEM FORMULATION

In this section, the kinematic model of the eccentric cam is derived and its LPV characteristic is illustrated.

A. Kinematic model of the eccentric cam

As illustrated in Fig. 1, the vertical positioning module is driven by an eccentric cam. The rotation of the cam pushes the follower pulley up and down, converting the input angle into vertical linear movement. From the geometry in Fig. 1, the actual vertical distance h between the rotation center of the cam and the center of the pulley can be calculated using the Law of Cosines:

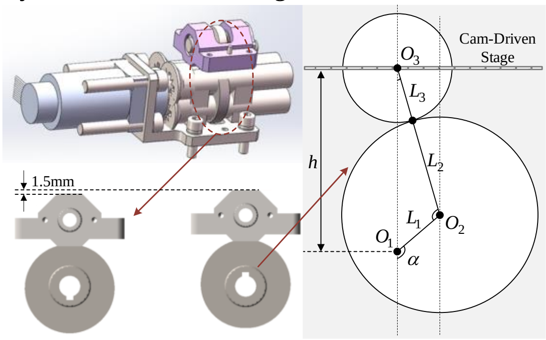

Fig.1 The actual structure and the kinematic principle of the cam-driven mechanism

$$(L_2 + L_3)^2 = h^2 + L_1^2 + 2hL_1 \cos\alpha \qquad (1)$$

where L_1 is the cam eccentricity, L_2 is the cam radius, L_3 is the pulley radius, and α is the input angle.

Solving Eq. (1) for h yields:

$$h = \sqrt{(L_2 + L_3)^2 - (L_1 \sin\alpha)^2} - L_1 \cos\alpha \qquad (2)$$

Defining the measurement zero point at the midpoint of the total stroke $h_{mid} = L_2 + L_3$, the actual vertical displacement $h_z = h - h_{mid}$.

B. LPV characteristic analysis

By differentiating h_z with respect to time, the transmission gain can be obtained:

$$\frac{\partial h_z}{\partial \alpha} = \frac{2h_z L_1 \sin\alpha}{2h_z + 2L_1 \cos\alpha} \qquad (3)$$

This derivation reveals the transmission gain between the rotational actuation and the measured vertical displacement varies nonlinearly with position, leading to a strong LPV characteristic. Furthermore, in practice, due to manufacturing tolerances and assembly misalignments, the true physical gain inevitably deviates from the nominal analytical model in Eq. (3). Consequently, both standard constant-gain feedforward and purely analytical inverse models are fundamentally inadequate, necessitating a data-driven tuning approach to dynamically compensate for this uncertain, position-dependent nonlinearity.

III. DESIGN OF PROPOSED METHOD

In this section, to compensate for the position-dependent nonlinearity gain, an LPV feedforward iterative tuning method is proposed.

A. Feedforward controller parameterization

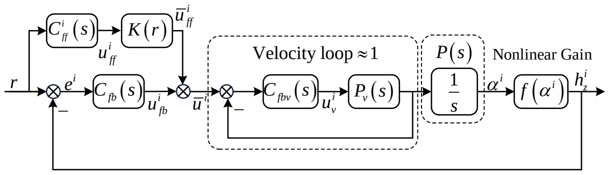

Fig. 2 Control block diagram of the cam motion system

As illustrated in Fig. 2, a position-velocity dual closed-loop control architecture is utilized. The velocity loop is designed with a sufficiently high bandwidth so that its closed-loop transfer function can be approximated as unity gain. Under this simplification, the control system operates effectively as a position tracking loop.

Based on the characteristics of the controlled object in Fig. 2, the reference-trajectory feedforward signal only needs to include velocity feedforward:

$$u_{ff}^* = \vartheta^* v \qquad (4)$$

where ϑ^* is the parameter, v is the reference velocity.

To compensate the unknown nonlinear mapping $f(\alpha^i)$ without accurate analytical model, $K^*(r)$ is approximated by polynomial basis expansion:

$$K^*(r) = \left(\sum_{j=0}^{n} \delta_j^* r^j \right) \qquad (5)$$

where δ_j^* is the parameter, r is the reference trajectory, j is the order of the polynomial.

Therefore, the ideal feedforward signal $\bar{u}_{ff}^*$ is:

$$\bar{u}_{ff}^* = u_{ff}^* K^*(r) = \boldsymbol{\Phi}\boldsymbol{\Theta}^* \qquad (6)$$

where $\boldsymbol{\Phi} = \left[v \; vr, \ldots, vr^n \right]$ and $\boldsymbol{\Theta} = \left[\theta_0^*, \theta_1^*, \ldots, \theta_n^* \right]^{\mathrm{T}}$ are basis function vector and parameter vector, respectively.

Unlike conventional feedforward controllers, the proposed structure explicitly embeds the interaction between position-dependent gain and motion dynamics.

B. Data-based LPV feedforward iterative tuning

The data-based feedforward iterative tuning method will be utilized to achieve $\boldsymbol{\Theta}^i \to \boldsymbol{\Theta}^*$. Assuming that $f^{-1}(\alpha^i) \approx K^i(r)\big|_{j=0}$, $P^i(s) = 1/C_{ff}^i(s)$, from Fig. 2, the error at the *i*-th trial can be expressed as:

$$e^i(s) = G^i(s)\left(\bar{u}_{ff}^*(s) - \bar{u}_{ff}^i(s) \right) \qquad (7)$$

where $G^i(s) \approx 1/\left(\theta_0^i s + C_{fb}(s)\right)$.

Based on Eq.(6), Eq.(8) can be further written as:

$$e^i(s) = G^i(s)\mathcal{L}(\boldsymbol{\Phi})\left(\boldsymbol{\Theta}^* - \boldsymbol{\Theta}^i\right) \tag{8}$$

where $\mathcal{L}(\cdot)$ is the Laplace transform.

Then, the data-based feedforward iterative tuning method can be developed as follows:

$$\boldsymbol{\Theta}^{i+1} = \boldsymbol{\Theta}^i + \boldsymbol{L}^i\boldsymbol{E}^i \tag{9}$$

where $\boldsymbol{L}^i = \left(\left(\boldsymbol{Z}^i\right)^{\mathrm{T}}\boldsymbol{Z}^i\right)^{-1}\left(\boldsymbol{Z}^i\right)^{\mathrm{T}}$, $\boldsymbol{Z}^i$ is the lifted form of $G^i(s)\mathcal{L}(\boldsymbol{\Phi})$, $\boldsymbol{E}^i$ is the lifted form of $e^i(t)$.

IV. EXPERIMENTAL RESULTS

This section verifies the effectiveness of the proposed method on a micro-positioning stage driven by three eccentric cams via comparative experiments.

A. Experimental setup

The experimental setup is shown in Fig. 3. As can be seen, the micro-positioning stage employs three identical cam units arranged at 120° intervals to achieve vertical motion. Assuming negligible parasitic rotational motion of the platform, the virtual Z-axis is driven by the three cams, and the equivalent vertical dynamics still preserve the fundamental LPV nonlinearity of a single cam. Therefore, subsequent experiments are conducted in the Z direction of the motion stage. The vertical displacement of each cam mechanism is measured by a grating scale. The sampling frequency of the control system is 5 kHz.

The order of basis functions in the feedforward controller is chosen as $n = 4$. The reference trajectory is shown in Fig. 4. The position-loop feedback controller is a PI controller with $K_p = 3\times10^6$ and $K_i = 1.5\times10^7$. To verify the effectiveness of the proposed method, a baseline using only velocity feedforward with iterative tuning (denoted as M1, which corresponds to setting $n = 0$ in Eq. (5)) is employed for comparison.

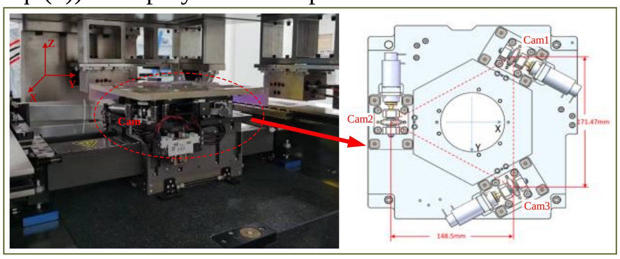

Fig. 3. Experimental set up

B. Experimental results

Fig. 5 shows the feedback controller output. If the system were LTI, this effort would remain constant during the constant-velocity phase. However, the significant drift explicitly confirms the varying transmission gain along the stroke, validating the inherent LPV characteristic of the system.Fig. 6 illustrates the evolution of tracking errors across iterative trials under the proposed method. It can be observed that the proposed method effectively reduces the tracking error, particularly during the acceleration and deceleration phases. Fig. 7 (a) shows the errors of the two methods at the 5th trial. Since M1 does not account for position-dependent gain variation, the proposed method yields smaller error (peak error 197nm) than M1 (peak error 264nm). Fig. 7 (b) presents the corresponding root-mean-square errors (RMSE). It is observed that both methods converge after the 2nd trial, but the proposed method consistently achieves lower errors.

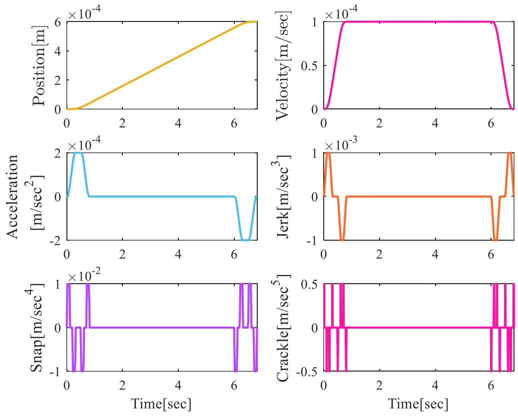

Fig.4 Reference trajectory

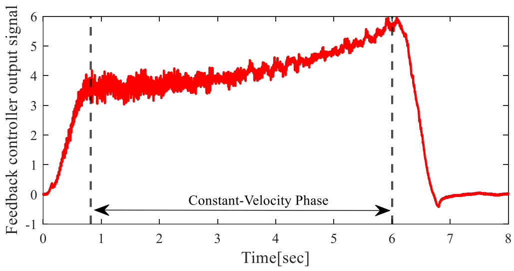

Fig. 5 Feedback controller output of the 0-th trial

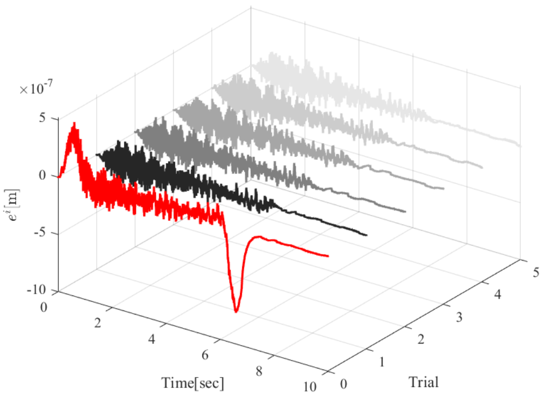

Fig.6 Error of the i-th trial under the proposed method

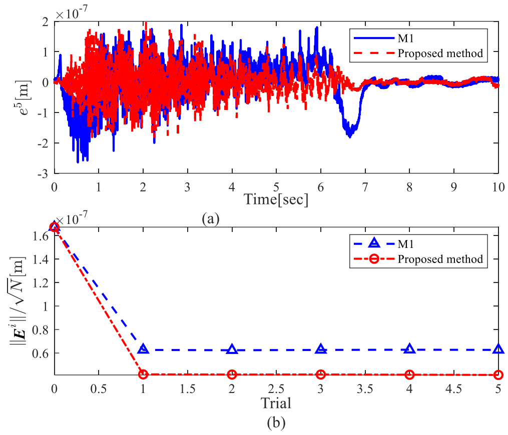

Fig. 7 Effectiveness of the proposed method: (a) Error of the 5th trial; (b) RMSE *of the i-th trial*

V. CONCLUSIONS AND OUTLOOK

This paper proposes a data-based LPV feedforward iterative tuning method for a cam-driven precision motion stage. with position-dependent nonlinear gain. By utilizing position-velocity coupled basis functions, the proposed approach effectively compensates for the inherent nonlinear gain without relying on accurate analytical models. Experimental results show that the proposed method significantly improves tracking accuracy compared with conventional feedforward methods.

Future work will explore algorithms to systematically determine the optimal set of basis functions in the absence of prior knowledge.

VI. ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China under Grant 52105546, and in part by the Heilongjiang Postdoctoral Fund under Grant LBH-Z21154 and Grant LBH-TZ2314.

REFERENCES

[1] Wang, Y; Zhu, J.; Pang, M.; Luo, J.; et al.: A stick-slip positioning stage robust to load variations, *IEEE/ASME Transactions on Mechatronics*, Vol. 21, No. 4, 2016, pp. 2165–2173.

[2] van Haren, M.; Blanken, L.; Oomen, T.: A kernel-based identification approach to LPV feedforward: With application to motion systems, *IFAC-PapersOnLine*, Vol. 56, No. 2, 2023, pp. 6063–6068.

[3] Huang, T.; Deng, P.; Zhang, W.; Xie, Z.; et al.: Polytopic LPV modeling and gain scheduling H∞ control of ball screw with position- and load-dependent variable dynamics, *Precision Engineering*, Vol. 87, 2024, pp. 1 – 10.

[4] Zhou, Z.; Gao, J.; Zhang, L.: A positioning and tracking performance–enhanced composite control algorithm for the macro–micro precision stage, *Actuators*, Vol. 13, No. 11, 2024, p. 433.

[5] Gatti, G.; Mundo, D.; Danieli, G.: A combined feedforward and feedback control strategy to improve the dynamic performance of cam-follower systems, 12th IFToMM World Congress, Besançon, France, June18-21, 2007.

[6] Zhuojuan, Y.; Wencheng, L.; Xiumin, D.: The influence of installation error on the motion of spatial cam mechanism and its simulation, *In IOP Conference Series: Materials Science and Engineering*, Vol. 392, No. 6, 2018, p. 062046.

[7] De Groote, W.; Van Hoecke, S.; Crevecoeur, G.: Physics-Based Neural Network Models for Prediction of Cam-Follower Dynamics Beyond Nominal Operations, IEEE/ASME Transactions on Mechatronics, Vol. 27, No. 4, 2022, pp. 2345–2355.

[8] Zhou, R.; Hu, C.; Wang, Z.; Zhu, Y.; et al.: Real-time iterative compensation control using plant-injection feedforward architecture with application to ultraprecision wafer stages, *IEEE Transactions on Industrial Informatics*, Vol. 20, No. 10, 2024, pp. 11708–11719.

[9] Chen, T.; Yuan, C.; Li, M.; Huang, X.; et al.: Data-based iterative tuning of feedforward control with disturbance compensation for dual-drive gantry stages in wafer inspection, *IEEE Transactions on Industrial Electronics*, Vol. 72, No. 9, 2025, pp. 9624-9634.

[10] Zhao, H.; Li, L.; Cui, N.; Liu, Y.; et al.: Iterative control tuning: With application to MIMO precision motion systems, *IEEE/ASME Transactions on Mechatronics*, Vol. 30, No. 4, 2024, pp. 2766–2777.