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LPV Feedforward Iterative Tuning for a Cam-Driven Precision Motion Stage

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***Abstract* – This paper proposes a data-based Linear Parameter-Varying (LPV) feedforward iterative tuning method for a cam-driven precision motion stage with position-dependent nonlinear gain. To overcome the limitations of standard Linear Time-Invariant (LTI) control methods, the proposed feedforward controller is parameterized using position-velocity coupled polynomial basis functions. Embedding high-order position variables into the velocity channel creates spatially varying basis functions that inherently capture the position-dependent LPV dynamics. Experiments on a cam-driven motion stage show that the proposed method converges within a small number of trials and reduces the peak tracking error from 264 nm to 197 nm compared with conventional velocity feedforward, corresponding to a reduction of 25.4%. These results demonstrate the benefit of explicitly incorporating the position-dependent transmission gain into data-based feedforward tuning.**

Keywords – Precision motion stage, LPV, Feedforward iterative tuning, Motion control.

I. INTRODUCTION

Cam-driven micro-positioning stages are widely used in precision manufacturing applications, such as semiconductor lithography, microscopy, and optical inspection, due to their compact structure, high stiffness, and excellent force density [1]. By converting continuous rotary motion into precise linear displacement, eccentric cam mechanisms provide an effective solution for high-load precision actuation. However, the input is applied in the rotational domain while the output is measured in the translational domain, resulting in a non-collocated input–output relationship and a nonlinear mapping between actuation and measurement.

From a control perspective, the resulting nonlinear mapping leads to a position-dependent gain, implying that the control system can no longer be characterized as Linear Time-Invariant (LTI), but rather exhibits Linear Parameter-Varying (LPV) characteristic [2,3]. Thus, LTI control methods struggle to maintain consistent stability margins and tracking performance throughout the entire stroke, leading to significant dynamic tracking errors [4].

To achieve higher tracking accuracy, feedforward control methods are widely adopted. In standard LTI systems, the feedforward controller is typically designed using constant gains proportional to the reference velocity and acceleration [5]. However, applying such a feedforward strategy directly to LPV systems is fundamentally inadequate, because the required amount of compensation varies with the operating point. Furthermore, although an accurate analytical model of the cam mechanism could in principle eliminate this nonlinearity, such an analytical model is often not fully applicable due to various installation deviations [6].

In practice, to avoid using accurate model information, data-driven methods are widely used [7-10]. While iterative learning control effectively suppresses repetitive disturbances, it lacks generalization to non-repetitive reference trajectories [8]. To address this limitation, feedforward iterative tuning can be used to optimize the parameters of a predefined feedforward controller [9,10]. Existing data-based feedforward iterative tuning methods mainly assume LTI basis structures with constant parameters, which limits their ability to compensate spatially varying dynamics.

In this paper, a data-based LPV feedforward iterative tuning method is developed for a cam-driven precision motion stage. By parameterizing the feedforward controller with position–velocity coupled polynomial basis functions, the position-dependent transmission gain can be effectively compensated without relying on an accurate analytical model. Unlike conventional velocity feedforward with constant gains, the feedforward compensation is adaptively scheduled according to the motion position. The main contributions of this work are summarized as follows:

1) The position-dependent nonlinear transmission gain of the cam-driven stage is explicitly formulated and interpreted as an LPV characteristic.

2) A position–velocity coupled basis structure is introduced to model spatially varying dynamics.

3) A data-driven iterative tuning strategy is developed to optimize the LPV feedforward parameters.

The remainder of this paper is organized as follows: Section II derives the kinematic model and illustrates the LPV characteristic of the stage. Section III details the LPV feedforward iterative tuning method. Section IV presents the experimental setup and comparative results. Finally, Section V concludes the paper.

II. PROBLEM FORMULATION

In this section, the kinematic model of the eccentric cam is derived and its LPV characteristic is illustrated.

A. Kinematic model of the eccentric cam

As illustrated in Fig. 1, the vertical positioning module is driven by an eccentric cam. The rotation of the cam pushes the follower pulley up and down, converting the input angle into vertical linear movement. From the geometry in Fig. 1, the actual vertical distance h between the rotation center of the cam and the center of the pulley can be calculated using the Law of Cosines:

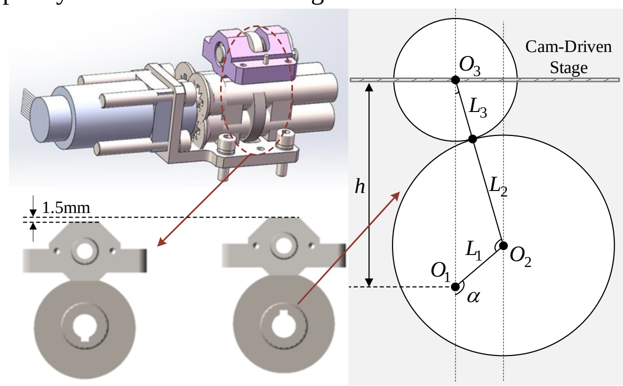

Fig. 1. The actual structure and the kinematic principle of the cam-driven mechanism

$$(L_2+L_3)^2 = h^2 + L_1^2 + 2hL_1\cos\alpha \tag{1}$$

where L_1 is the cam eccentricity, L_2 is the cam radius, L_3 is the pulley radius, and α is the input angle.

Solving Eq. (1) for h yields:

$$h = \sqrt{(L_2+L_3)^2 - (L_1\sin\alpha)^2} - L_1\cos\alpha \tag{2}$$

Defining the measurement zero point at the midpoint of the total stroke $h_{mid} = L_2 + L_3$, the actual vertical displacement $h_z = h - h_{mid}$.

B. LPV characteristic analysis

By differentiating h_z with respect to the input angle α, the transmission gain can be obtained:

$$\frac{\partial h_z}{\partial \alpha} = \frac{2hL_1\sin\alpha}{2h+2L_1\cos\alpha} \tag{3}$$

This derivation reveals the transmission gain between the rotational actuation and the measured vertical displacement varies nonlinearly with position, leading to a strong LPV characteristic. Furthermore, in practice, due to manufacturing tolerances and assembly misalignments, the true physical gain inevitably deviates from the nominal analytical model in Eq. (3). Consequently, both standard constant-gain feedforward and purely analytical inverse models are fundamentally inadequate, necessitating a data-driven tuning approach to dynamically compensate for this uncertain, position-dependent nonlinearity.

III. DESIGN OF PROPOSED METHOD

In this section, to compensate for the position-dependent nonlinear gain, an LPV feedforward iterative tuning method is proposed.

A. Feedforward controller parameterization

As illustrated in Fig. 2, a position-velocity dual closed-loop control architecture is utilized. r is the reference trajectory, and $e^i = r - h_z^i$ is the tracking error at the i-th trial, where h_z^i is the measured vertical displacement. $P(s)$ and $P_v(s)$ are the position-loop controlled plant and velocity-loop controlled plant, respectively. $C_{fb}(s)$ and $C_{fbv}(s)$ are the position-loop feedback controller and velocity-loop feedback controller, respectively. $C_{ff}^i(s)$ is the reference-trajectory feedforward controller, $K^i(r)$ is utilized to compensate for the position-dependent nonlinear transmission gain $f(\alpha^i)$.

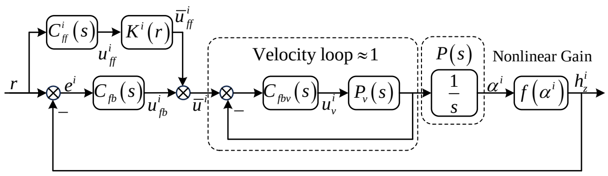

Fig. 2. Control block diagram of the cam motion system

The velocity-loop is designed with a sufficiently high bandwidth so that its closed-loop transfer function can be approximated as unity gain. Under this simplification, the control system operates effectively as a position tracking loop. Based on the characteristics of the controlled object in Fig. 2, the ideal reference-trajectory feedforward signal only needs to include velocity feedforward:

$$u_{ff}^* = \vartheta^* v \tag{4}$$

where ϑ^* is the corresponding parameter, v is the reference velocity.

Considering that the feedforward signal remains proportional to motion velocity while the LPV characteristic varies smoothly with position, polynomial basis functions provide a compact approximation of the nonlinear gain. To compensate the unknown nonlinear gain $f\left(\alpha^i\right)$ without an accurate analytical model, the ideal $K^*\left(r\right)$ is approximated by polynomial basis expansion:

$$K^*\left(r\right)=\left(\sum_{j=0}^{n}\delta_j^* r^j\right) \tag{5}$$

where δ_j^* is the corresponding parameter, j is the order of the polynomial.

Define the combined parameter as $\theta_j^*=\vartheta^*\delta_j^*$, therefore, defining the ideal feedforward signal $\bar{u}_{ff}^*$ as:

$$\bar{u}_{ff}^*=u_{ff}^* K^*\left(r\right)=\boldsymbol{\Phi}\boldsymbol{\Theta}^* \tag{6}$$

where $\boldsymbol{\Phi}=\left[v, vr, \ldots, vr^n\right]$ and $\boldsymbol{\Theta}^*=\left[\theta_0^*, \theta_1^*, \ldots, \theta_n^*\right]^{\mathrm{T}}$ are the basis function vector and corresponding parameter vector, respectively.

Unlike conventional feedforward controllers with constant gains, the proposed structure explicitly embeds the interaction between position-dependent gain and motion dynamics. As a result, the feedforward controller can adapt its compensation effort according to the instantaneous motion position, enabling effective LPV compensation.

B. Data-based LPV feedforward iterative tuning

The data-based feedforward iterative tuning method will be utilized to achieve $\boldsymbol{\Theta}^i\rightarrow\boldsymbol{\Theta}^*$. Assuming that $f^{-1}\left(\alpha^i\right)\approx K^i\left(r\right)\big|_{j=0}$, $P^i\left(s\right)=1/C_{ff}^i\left(s\right)$, from Fig. 2, the following relationship can be obtained:

$$\begin{cases} u_{fb}^i\left(s\right)=C_{fb}\left(s\right)e^i\left(s\right) \\ u_{ff}^i\left(s\right)=C_{ff}^i\left(s\right)r\left(s\right) \\ \bar{u}_{ff}^i\left(s\right)=K^i\left(r\right)u_{ff}^i\left(s\right) \end{cases} \tag{7}$$

The actual system output is:

$$\begin{aligned} h_z^i &= P\left(s\right)f\left(\alpha^i\right)\left[u_{fb}^i\left(s\right)+\bar{u}_{ff}^i\left(s\right)\right] \\ &= P\left(s\right)f\left(\alpha^i\right)\left[C_{fb}\left(s\right)e^i\left(s\right)+\bar{u}_{ff}^i\left(s\right)\right] \end{aligned} \tag{8}$$

The ideal feedforward $\bar{u}_{ff}^*\left(s\right)$ can achieve perfect tracking, so it should satisfy:

$$r\left(s\right)=P\left(s\right)f\left(\alpha^i\right)\bar{u}_{ff}^*\left(s\right) \tag{9}$$

According to the error definition $e^i(s)=r\left(s\right)-h_z^i\left(s\right)$, the following relationship can be obtained:

$$\begin{aligned} e^i(s) &= P\left(s\right)f\left(\alpha^i\right)\left(\bar{u}_{ff}^*\left(s\right)-\bar{u}_{ff}^i\left(s\right)\right) \\ &\quad -P\left(s\right)f\left(\alpha^i\right)C_{fb}\left(s\right)e^i\left(s\right) \end{aligned} \tag{10}$$

By rearranging, one can obtain:

$$e^i(s)=G^i\left(s\right)\left(\bar{u}_{ff}^*\left(s\right)-\bar{u}_{ff}^i\left(s\right)\right) \tag{11}$$

where $G^i(s)\approx 1\big/\left(\theta_0^i s+C_{fb}(s)\right)$.

Based on Eq.(6), Eq.(11) can be further written as:

$$e^i(s)=G^i\left(s\right)\mathcal{L}\left(\boldsymbol{\Phi}\right)\left(\boldsymbol{\Theta}^*-\boldsymbol{\Theta}^i\right) \tag{12}$$

where $\mathcal{L}\left(\cdot\right)$ means the Laplace transform.

Then, the data-based feedforward iterative tuning method can be developed as follows:

$$\boldsymbol{\Theta}^{i+1}=\boldsymbol{\Theta}^i+\boldsymbol{L}^i\boldsymbol{E}^i \tag{13}$$

where $\boldsymbol{L}^i=\left(\left(\boldsymbol{Z}^i\right)^{\mathrm{T}}\boldsymbol{Z}^i\right)^{-1}\left(\boldsymbol{Z}^i\right)^{\mathrm{T}}$, $\boldsymbol{Z}^i$ is the lifted form of $G^i\left(s\right)\mathcal{L}\left(\boldsymbol{\Phi}\right)$, $\boldsymbol{E}^i$ is the lifted form of $e^i\left(t\right)$.

The implementation procedure of the proposed data-based LPV feedforward iterative tuning method is summarized in Table 1. At each trial, the measured tracking error is utilized to update the coefficients of the position-velocity coupled basis functions. The iteration is terminated when the maximum number of trials is reached.

C. Parameter Convergence Analysis

Define the parameter estimation error at the *i*-th trial as

$$\tilde{\boldsymbol{\Theta}}^i=\boldsymbol{\Theta}^*-\boldsymbol{\Theta}^i \tag{14}$$

According to Eq. (12), the lifted tracking error can be written as：

$$\boldsymbol{E}^i=\boldsymbol{Z}^i\tilde{\boldsymbol{\Theta}}^i \tag{15}$$

Substituting the parameter update law in Eq. (13) gives

$$\begin{aligned}\tilde{\boldsymbol{\Theta}}^{i+1} &= \boldsymbol{\Theta}^{*} - \boldsymbol{\Theta}^{i+1} \\ &= \boldsymbol{\Theta}^{*} - \boldsymbol{\Theta}^{i} - \boldsymbol{L}^{i}\boldsymbol{E}^{i} \\ &= \left(\boldsymbol{I} - \boldsymbol{L}^{i}\boldsymbol{Z}^{i}\right)\tilde{\boldsymbol{\Theta}}^{i}\end{aligned} \tag{16}$$

If $\boldsymbol{Z}^{i}$ has full column rank, the following is satisfied:

$$\boldsymbol{L}^{i}\boldsymbol{Z}^{i} = \boldsymbol{I} \tag{17}$$

Under ideal noise-free and perfectly modeled conditions, the parameter error can theoretically be eliminated after one update, i.e., $\tilde{\boldsymbol{\Theta}}^{i+1} = \boldsymbol{0}$.

In practice, the measured tracking error contains measurement noise, unmodeled dynamics, and trial-to-trial variations. Therefore, the lifted tracking error can be written as:

$$\boldsymbol{E}^{i} = \boldsymbol{Z}^{i}\tilde{\boldsymbol{\Theta}}^{i} + w^{i} \tag{18}$$

where w^{i} represents the lumped uncertainty. The corresponding parameter error becomes:

$$\tilde{\boldsymbol{\Theta}}^{i+1} = \left(\boldsymbol{I} - \boldsymbol{L}^{i}\boldsymbol{Z}^{i}\right)\tilde{\boldsymbol{\Theta}}^{i} - \boldsymbol{L}^{i}w^{i} \tag{19}$$

If $\left\|\boldsymbol{I} - \boldsymbol{L}^{i}\boldsymbol{Z}^{i}\right\| < 1$ and w^{i} is bounded, the parameter error remains bounded and approaches a neighborhood of zero. The size of this neighborhood depends on the noise and modeling uncertainty.

Table 1. The data-based LPV feedforward iterative tuning algorithm

Algorithm. Data-Based LPV Feedforward Iterative Tuning

Input: Reference position r, reference velocity v, polynomial order n, maximum number of trials $i_{\max}$.

Output: Tuned LPV feedforward parameter vector $\boldsymbol{\Theta}^{i}$.

1. Construct the position-velocity coupled basis function: $\boldsymbol{\Phi}$.
2. Initialize $\boldsymbol{\Theta}^{0} \leftarrow 0$ and $i \leftarrow 0$.

for $i < i_{\max}$

3. Generate the feedforward input $\bar{u}_{ff}^{i} = \boldsymbol{\Phi}\boldsymbol{\Theta}^{i}$.
4. Perform the *i*-th motion trial using $\bar{u}_{ff}^{i}$.
5. Measure the vertical displacement and calculate e^{i}.
6. Construct $\boldsymbol{E}^{i}, G^{i}(s), \boldsymbol{Z}^{i}$ and $\boldsymbol{L}^{i}$.
7. Update the parameters $\boldsymbol{\Theta}^{i+1}$ using Eq. (13).
8. $i \leftarrow i + 1$.

return $\boldsymbol{\Theta}^{i}$.

end

IV. EXPERIMENTAL RESULTS

This section verifies the effectiveness of the proposed method on a micro-positioning stage driven by three eccentric cams via comparative experiments.

A. Experimental setup

As shown in Fig. 3, the experimental platform is a complex six-degree-of-freedom (6-DOF) wafer stage. Specifically, the three horizontal DOFs are directly driven by linear motors, whereas the three vertical DOFs are actuated by rotary motors coupled with eccentric cam mechanisms. Focusing on the vertical micro-positioning stage, it employs three identical cam units arranged at 120° intervals to achieve precise vertical motion. Assuming negligible parasitic rotational motion of the platform, the equivalent virtual logical Z-axis is synchronously driven by these three cams. The equivalent vertical dynamics inherently preserves the fundamental position-dependent LPV nonlinearity of a single cam. Therefore, the subsequent validation experiments are conducted exclusively in the Z-axis direction of the motion stage. The vertical displacement of each cam mechanism is measured by a grating scale. The system sampling frequency is 5 kHz.

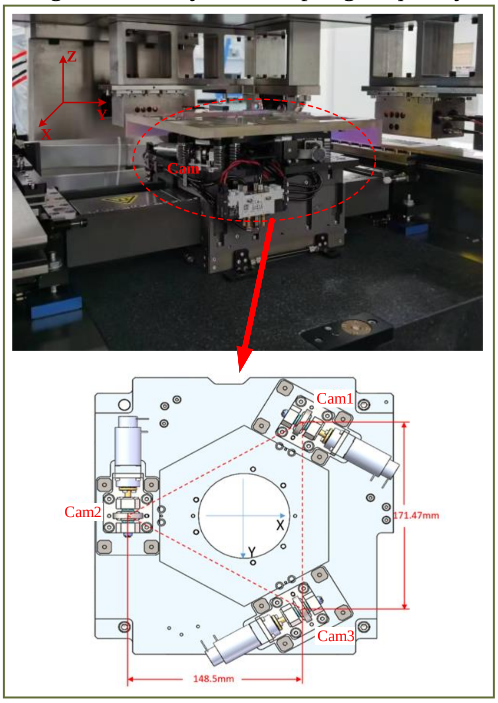

Fig. 3. Experimental setup

The position-loop feedback controller of logical Z-axis adopts a Proportional-Integral (PI) structure:

$$C_{fb}(s) = K_p + K_i \frac{1}{s} \tag{20}$$

where the corresponding parameters are set as $K_p = 3\times10^{6}$ and $K_i = 1.5\times10^{7}$.

The order of basis functions in the feedforward controller is chosen as $n = 4$. The reference trajectory is a 5th-order S-curve and the details are shown in Fig. 4. Specifically, to verify the effectiveness of the proposed method, a method using only velocity feedforward with iterative tuning (which is denoted as M1 and corresponds to $n = 0$ in Eq. (5)) is employed for comparison.

B. Experimental results

Fig. 5 shows the position-loop feedback control input during the 0-th trial. During the constant-velocity phase, the feedback input increases continuously with the position instead of remaining constant. This position-dependent variation demonstrates that the rotational-to-translational transmission gain changes over the motion stroke, thereby confirming the LPV characteristic of the cam-driven motion stage.

Fig. 6 presents the evolution of the tracking error over successive trials using the proposed method. The large transient errors observed in the initial trial are substantially reduced after the first update, and only minor changes occur during the subsequent trials, indicating fast and stable convergence. Fig. 7 further compares the tracking errors obtained by M1 and the proposed method at the 5-th trial. As shown in Table 2, the proposed method reduces the peak tracking error from 264 nm to 197 nm, corresponding to an improvement of 25.4%. In particular, the errors near the acceleration and deceleration phases are more effectively suppressed by the proposed method.

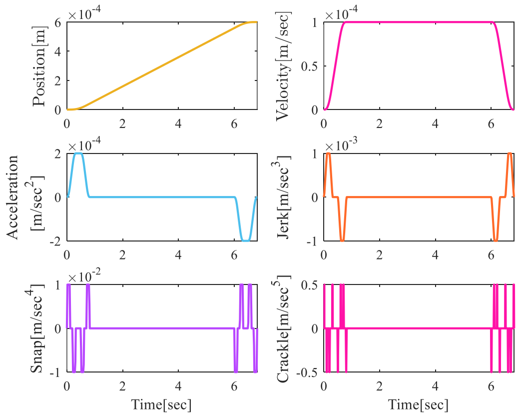

Fig. 4. Reference trajectory

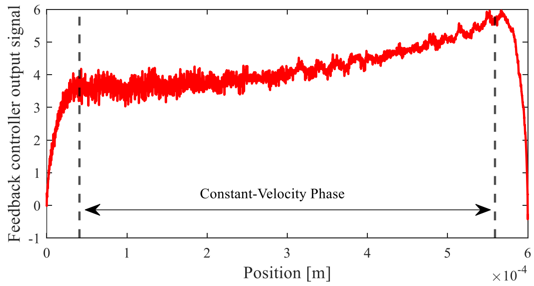

Fig. 5. Feedback controller output of the 0-th trial

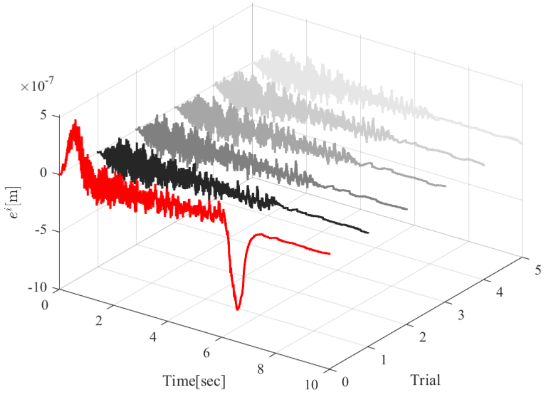

Fig. 6. Error of the i-th trial under the proposed method

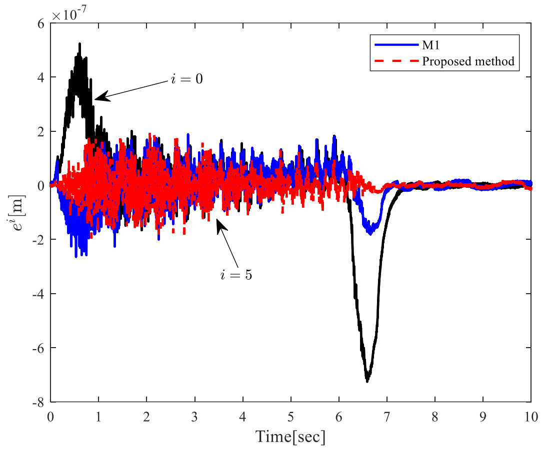

Fig. 7. Effectiveness of the proposed method: Error of the 5-th trial

Table 2. Quantitative comparison at the 5-th trial

Method	Peak error [nm]	RMSE [nm]
M1	264	63
Proposed Method	197	41

The learned velocity feedforward gains are shown in Fig. 8. M1 produces a constant gain ($K^5(r)$ with $n = 0$) over the complete stroke, whereas the gain generated by the proposed method ($K^5(r)$ with $n = 4$) increases continuously with position. continuously with position. This result confirms that the position-velocity coupled polynomial basis successfully captures the spatial variation of the transmission gain and provides position-dependent feedforward compensation. As shown in Fig. 9, both methods obtain most of their error reduction after the first update. As shown in Table 2, the root-mean-square error (RMSE) values of M1 and the proposed method at the 5-th trial are 63 nm and 41 nm, respectively, demonstrating an additional RMSE reduction of 34.9% through LPV compensation.

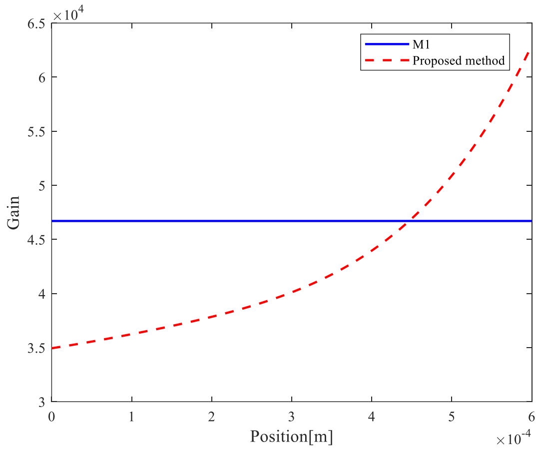

Fig. 8. Effectiveness of the proposed method: Velocity feedforward gain $K^5(r)$

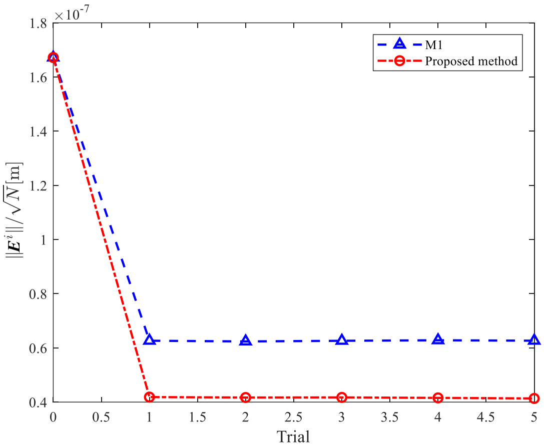

Fig. 9. Effectiveness of the proposed method: RMSE of the i-th trial

V. CONCLUSIONS AND OUTLOOK

This paper proposes a data-based LPV feedforward iterative tuning method for a cam-driven precision motion stage with position-dependent nonlinear gain. By utilizing position-velocity coupled basis functions, the proposed method effectively compensates for the inherent nonlinear gain without relying on an accurate analytical model. Experimental results show that the proposed method significantly improves tracking accuracy compared with conventional feedforward methods.

Future work will explore algorithms to systematically determine the optimal set of basis functions in the absence of prior knowledge.

VI. ACKNOWLEDGMENTS

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